

Effective Field Theory in AdS: Continuum Regime, Soft Bombs, and IR Emergence

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Abstract

We consider an effective theory of interacting matter fields in a slice of five-dimensional Lorentzian AdS at arbitrary energies. Using dimensional and quantitative arguments, we determine there exists a transition scale at which the IR brane effectively leaves the theory. In the presence of bulk interactions there are thus two qualitatively different regimes: Kaluza–Klein and continuum. Bulk interactions correspond to finite N in the dual gauge theory; we find the transition scale is consistent with the cutoff scale of a large- N EFT of glueballs. We then study the cascade decays of a scalar field with cubic self-interactions, with a focus on the continuum regime. We find that the cascade decay progresses slowly towards the IR region and gives rise to soft spherical final states, in accordance with former results from both gravity and CFT. We identify a recursion relation between integrated squared amplitudes of different leg numbers and thus evaluate the total rate. We find that cascade decays in the continuum regime are exponentially suppressed. This feature completes the picture of the IR brane as an emergent sector as seen from the UV brane. We briefly discuss some implications for holographic dark sector models.

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1 Introduction

There is overwhelming evidence that gauge theories with large 't Hooft coupling exhibit vastly different behaviour than weakly-coupled gauge theories, see e.g. [1]. Though direct calculability is lost in this limit, the AdS/CFT correspondence dictates that the gauge theory is dual to a weakly coupled string theory with a curved extra dimension [2–5]. At sufficiently large 't Hooft coupling, string states in the gravity dual are heavy such that the 5D theory consists of an effective field theory living in an AdS_5 background, see e.g. [6–11].

In the original correspondence, the 4D gauge theory, $\mathcal{N} = 4$ super-Yang–Mills, is perfectly conformal. This corresponds to an exactly AdS_5 metric in the gravity dual. Variations of the duality in which the metric is deformed/truncated in the IR region of AdS have also been proposed [12–22]). The IR departure from AdS in these models can lead to a discrete tower of states resembling a hadron spectrum in QCD. For the present work it is enough to focus on the simplest incarnation of this idea, the “slice of AdS”. We expect that our conclusions generalize to other IR departures from AdS, see Section 6.4.

Unlike QCD—which has small 't Hooft coupling—cascades of radiation at large 't Hooft coupling do not form jets because there is no reason for soft or collinear phase space configurations to be preferred. Instead, there is convincing evidence that cascades instead tend to be democratic in momentum splitting and give rise to spherical events with a large number of low-momentum final states. In this paper, we refer to this strongly-coupled analog of jets as *soft bombs*, though they are also referred to as “spherical events” or “jets at strong coupling”. Many studies of soft bombs have been done on both the gauge theory side and on the gravity side, for example the seminal paper by Hofman and Maldacena [23] and related works [24–28]. A study of field-theoretical soft bombs in AdS_5 has been done in [29], in which narrow Kaluza–Klein (KK) modes were considered. In this work we show that the regime of narrow KK modes is only one aspect of the full picture. In fact, the narrow KK mode regime may even be essentially absent of the 5D theory.

We briefly sketch the energy regimes of field theory in a slice of 5D AdS, as obtained in this work. The fundamental scales fixed by geometry are the AdS curvature, k , and the IR brane position, $1/\mu$. The Kaluza–Klein scale is $\mu \ll k$ and represents the mass gap in the dual gauge theory. In the presence of interactions, the theory has a 5D cutoff Λ and a transition scale $\tilde{\Lambda}$ that is explained below. These have a hierarchy $\Lambda > k > \tilde{\Lambda} > \mu$ that define four different energy regimes:

- 4D regime, $E < \mu$. In this limit, Kaluza–Klein modes are integrated out and only sufficiently light 4D modes such as gauge or Goldstone bosons remain in the spectrum.
- Kaluza–Klein regime, $\mu < E < \tilde{\Lambda}$. The theory in this regime has a tower of regularly spaced narrow resonances. This window of energy is favored by the large- N limit of the dual gauge theory where the resonances are identified as narrow glueballs.
- Continuum regime, $\tilde{\Lambda} < E < k$. In this regime, the effective theory breaks down in the IR region of AdS. Quantum corrections mix the KK modes and merge them into a continuum. An observer on the UV brane effectively sees pure AdS. The theory can equivalently be described by a holographic CFT model with no mass gap.
- Flat space regime, $k < E < \Lambda$. Here the curvature of AdS becomes negligible, and KK modes from any other compact dimensions appear. No simple CFT dual is expected in this regime.

The presence of the distinct Kaluza–Klein and continuum regimes can be deduced qualitatively from [30, 31]. A quantitative description of the transition and the typical scale $\tilde{\Lambda}$, however, is more

subtle. Interactions (even 5D gravity) resolve the KK poles through bulk quantum corrections to the self-energy. One must account for these corrections to observe the transition between narrow KK modes and continuum [32]. Across this transition, the bulk correlators in the continuum regime effectively lose contact with the IR brane as the effective theory breaks down in that region of position–momentum. We say that the IR brane *effectively emerges* for bulk correlators as their energy is decreased through this KK–continuum transition.

One puzzle is whether cascade decays into soft bombs may challenge this picture of effective emergence. Cascade decays can split the energy of individual excitations across many offspring states. Thus a cascade can convert a single state in the continuum regime into many states in the KK regime. The soft bomb naïvely appears to be a way for a bulk field to propagate information to the IR brane even when the initial excitation is in a regime where it is not sensitive to the IR brane. The picture of effectively emergent IR brane physics thus depends on a careful understanding of soft bomb events from bulk decays.

In this work, we establish the existence of a continuum regime in the presence of interactions and study soft bombs events in this regime. There are multiple motivations for such a study:

- The earlier work on soft bombs in the Kaluza–Klein regime [29] does not apply in the continuum regime because the effective theory breaks down in the IR region of AdS. KK modes are thus not appropriate degrees of freedom. We thus investigate whether events are indeed spherical and soft in the continuum regime. This also serves as a check of the soft bomb picture in the CFT dual.
- In addition to the kinematic considerations, we calculate occurrence probabilities for soft bomb events. To the best of our knowledge, such a calculation has not been presented in the literature.
- Understanding the KK–continuum transition and the soft bomb rate allows us to complete the picture of the emergence of the IR brane. Without soft bomb rates, it remains unclear whether the theory can actually be described by a high-energy effective theory with no IR brane in the continuum regime.
- Both IR brane emergence and the properties of soft bombs have phenomenological implications for models of physics beyond the Standard Model that involve a strongly-coupled hidden sector with an AdS dual. This holographic dark sector scenario has been recently put presented in [33, 34], see also [35–40] for earlier and related attempts.

This paper is organized as follows. Section 2 establishes the basic five-dimensional formalism in a slice of AdS. In particular, we present the classical propagator for a scalar field in mixed position–momentum space. Interactions in the bulk of AdS play a central role in our study. Section 3 provides the necessary tools for dimensional analysis at strong coupling. We establish the relation between bulk matter interactions and the large- N expansion of the dual CFT. In Section 4, we dress the propagator with quantum corrections. The imaginary part of the self-energy induces distinct KK and continuum regimes. The transition scale is understood both qualitatively from the viewpoint of effective theory validity and from the viewpoint of the opacity of the IR region resulting from the dressing of the propagator by bulk fields. In Section 5 we identify a recursion relation that relates the continuum-regime cascade decay rates with arbitrary number of legs. Section 6 presents the general picture of soft bomb events in the continuum regime. The total rate for these events is exponentially suppressed and we discuss implications for IR brane emergence and holographic dark sector scenarios.

2 A Bulk Scalar in a Slice of AdS

In studies of the gravity-scalar system, a general ansatz for the metric preserving the 4D Poincaré invariance is

$$ds^2 = g_{MN} dX^M dX^N = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (2.1)$$

where $\eta_{\mu\nu}$ is the 3+1-dimensional Minkowski metric with $(+, -, -, -)$ signature. Such system appears in certain 5D supergravities, see *e.g.* [41]. The metric can depart from AdS and develop a singularity at large y , beyond which spacetime ends, see *e.g.* [12–14, 17, 19, 20]. In other classes of models, an IR brane truncating the y coordinate is explicitly included.

For the focus of this paper, it is enough to focus on the simplest setup of a “slice of AdS” for which the metric remains exactly AdS everywhere. We briefly comment on more general cases in Section 6.4. Using the conformally flat coordinates $z = e^{ky}/k$, the metric is

$$ds^2 = g_{MN} dX^M dX^N = (kz)^{-2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2). \quad (2.2)$$

Space is truncated at endpoints

$$z_{\text{UV}} = k^{-1} \quad \text{and} \quad z_{\text{IR}} = \mu^{-1} > z_{\text{UV}}, \quad (2.3)$$

which correspond to the positions of a UV and IR brane, respectively.

2.1 Action

A generic effective theory on this background involves gravitons and matter fields of different spins. In this manuscript we focus on the case of a scalar field Φ with non-derivative, cubic interactions. We expect that the results of this study generalize readily to any other type of field. The action for this field is

$$S = \int d^5 X \sqrt{g} \left(\frac{1}{2} \nabla_M \Phi \nabla^M \Phi - \frac{1}{2} m_\Phi^2 \Phi^2 + \frac{1}{3!} \lambda \Phi^3 \right) + S_{\text{UV}} + S_{\text{IR}} + \dots \quad (2.4)$$

where we explicitly write the kinetic, mass and interaction terms. The ellipses denote additional contributions from gravity and higher-dimensional operators that are suppressed by powers of the effective theory’s cutoff. A convenient parameterization of the scalar mass is

$$m_\Phi^2 \equiv (\alpha^2 - 4)k^2. \quad (2.5)$$

The Breitenlonher-Friedmann bound requires $\alpha \geq 0$ [42, 43]. In this work we routinely take α to be non-integer. The actions S_{UV} and S_{IR} encode brane-localized operators. These can include mass terms for the scalar which are conveniently parameterized with respect to dimensionless parameters b_{UV} and b_{IR} as (see, *e.g.* [44]),

$$S_{\text{UV}} + S_{\text{IR}} \supset \frac{1}{2} \int d^5 X \sqrt{\bar{g}} [(\alpha - 2 - b_{\text{UV}})k\delta(z - z_{\text{UV}}) - (\alpha - 2 + b_{\text{IR}})k\delta(z - z_{\text{IR}})] \Phi^2. \quad (2.6)$$

We leave these parameters unspecified and simply assume that $b_{\text{UV}} \neq 0$. There is a special mode in the spectrum with mass $\sim b_{\text{UV}}k$. For b_{UV} sufficiently small, this mode may affect the physical processes studied here. We assume this special mode is heavy such that it is irrelevant in our analysis. $\bar{g}_{\mu\nu}$ is the induced metric on the brane so that $\sqrt{\bar{g}} = (kz)^{-4}$. Other degrees of freedom

may be localized on the brane and interact with Φ .¹ In the context of our analysis, such brane modes provide asymptotic states for the bulk scattering amplitudes.

2.2 The Scalar Propagator

The classical equation of motion obtained by varying the bulk action for the scalar field, Φ , is

$$D\Phi \equiv \frac{1}{\sqrt{g}}\partial_M(g^{MN}\sqrt{g}\partial_N\Phi) + m_\Phi^2\Phi = 0. \quad (2.7)$$

The Feynman propagator is the Green's function of the D operator,

$$D_X\Delta(X, X') = \frac{-i}{\sqrt{g}}\delta^{(5)}(X - X'). \quad (2.8)$$

In mixed position-momentum space it is [45]

$$\Delta_p(z, z') = i \frac{\pi k^3 (zz')^2}{2} \frac{\left[\tilde{Y}_\alpha^{UV} J_\alpha(pz_{<}) - \tilde{J}_\alpha^{UV} Y_\alpha(pz_{<}) \right] \left[\tilde{Y}_\alpha^{IR} J_\alpha(pz_{>}) - \tilde{J}_\alpha^{IR} Y_\alpha(pz_{>}) \right]}{\tilde{J}_\alpha^{UV} \tilde{Y}_\alpha^{IR} - \tilde{Y}_\alpha^{UV} \tilde{J}_\alpha^{IR}}, \quad (2.9)$$

where $z_{<,>}$ is the lesser/greater of the endpoints z and z' . We use the Minkowski norm of the 4-momentum $p = \sqrt{\eta_{\mu\nu}p^\mu p^\nu}$, which is real (imaginary) for timelike (spacelike) four-momentum, p^μ . The p -dependent quantities $\tilde{J}^{UV,IR}$ are

$$\tilde{J}_\alpha^{UV} = \frac{p}{k} J_{\alpha-1}\left(\frac{p}{k}\right) - b_{UV} J_\alpha\left(\frac{p}{k}\right) \quad \tilde{J}_\alpha^{IR} = \frac{p}{\mu} J_{\alpha-1}\left(\frac{p}{\mu}\right) + b_{IR} J_\alpha\left(\frac{p}{\mu}\right), \quad (2.10)$$

with similar definitions for $\tilde{Y}^{UV,IR}$.

For timelike momentum, the propagator (2.9) has poles set by the zeros of the denominator. This propagator can always be written formally as an infinite sum over 4D poles. Let us introduce the matrix notation

$$\mathbf{f}(z) = [f_n(z)] \quad \mathbf{D} = \left[\frac{\delta_{np}}{q^2 - m_n^2} \right], \quad (2.11)$$

where \mathbf{f} is a one-dimensional infinite vector and \mathbf{D} is an infinite diagonal matrix indexed by the Kaluza-Klein (KK) numbers n and p . The propagator in the Kaluza-Klein representation is

$$\Delta_p(z, z') = i \mathbf{f}(z) \cdot \mathbf{D} \cdot \mathbf{f}(z'). \quad (2.12)$$

Amplitude calculations often feature sums over KK modes. We can represent these sums as contour integrals [32],

$$\sum_{n=0}^{\tilde{n}} U(m_n) f_n(z) f_n(z') = -\frac{1}{2\pi} \oint_{\mathcal{C}[\tilde{n}]} dq^2 U(q^2) \Delta_q(z, z'), \quad (2.13)$$

where the contour $\mathcal{C}[\tilde{n}]$ in momentum space encloses the first \tilde{n} poles. U can be any function that does not obstruct the contour with singularities. The identity (2.13) is a useful link between the KK and closed form representations of the propagator.

¹There are hints that a brane-localized degree of freedom always arises from a bulk field and is thus necessarily accompanied by a tower of Kaluza-Klein modes [45]. This tower can be decoupled from the brane so that it is consistent to consider only the brane-localized mode.

3 Interactions: Dimensional Analysis and Large N

A key ingredient of our study is the magnitude of the couplings of the bulk scalar from an effective field theory (EFT) perspective. In the presence of interactions, a five-dimensional theory is understood to be an EFT with some ultraviolet cutoff Λ beyond which the EFT becomes strongly coupled. This cutoff is tied to the strength of interactions through dimensional analysis in the strong coupling limit through so-called naïve dimensional analysis (NDA) [46–50]; see e.g. [44] for a pedagogical introduction of NDA to 5D theories. The crux of this analysis is to compare amplitudes of different loop order or involving higher dimensional operators. Let us define the loop factors

$$\ell_5 = 24\pi^3 \quad \text{and} \quad \ell_4 = 16\pi^2 . \quad (3.1)$$

3.1 Gravitational Interactions

The interactions of the graviton in AdS is controlled by the dimensionless coupling

$$\kappa = \frac{k}{M_{\text{Pl}}} . \quad (3.2)$$

The reduced 4D and 5D Planck masses are related by $M_5^3 = M_{\text{Pl}}^2 k$. By NDA, the cutoff in the gravity sector

$$\Lambda_{\text{grav}}^3 = \ell_5 M_5^3 = \ell_5 \kappa M_{\text{Pl}}^3 . \quad (3.3)$$

In order to keep higher order gravity terms under control, κ should be at most $\mathcal{O}(1)$ [49, 51].

The gravity cutoff Λ_{grav} is sometimes taken as a universal scale setting the strength of all interactions in the effective Lagrangian. However, in the EFT the typical strength of interactions in various sectors can in principle be different with different strong coupling scales. Strongly-interacting matter cannot influence the strength of gravity, which is protected by diffeomorphism invariance and set by the background geometry. In particular, matter interactions are at least as strong as gravity. The strong coupling scale of pure matter interactions can thus be lower than Λ_{grav} . Notice that gravity can even be removed, $M_{\text{Pl}} \rightarrow \infty$, while the matter cutoff remains unchanged.²

3.2 Matter Interactions

We assume that a universal cutoff Λ sets the strength of interactions in the matter sector of or theory. To make this connection manifest in D -dimensions, one writes the fundamental action in terms of dimensionless fields $\hat{\Phi}$ with ℓ_D factored out [44, 49]:

$$S_D = \frac{N_s \Lambda^D}{\ell_D} \int d^D X \hat{\mathcal{L}} \left[\hat{\Phi}, \partial/\Lambda \right] . \quad (3.4)$$

N_s counts the number of species in the Lagrangian; for the present study we set $N_s = 1$. NDA states that an $\mathcal{O}(1)$ coupling in $\hat{\mathcal{L}}$ corresponds to a strong interaction strength. The dimensionful

²In a UV completion, the Λ , Λ_{grav} scales would likely be correlated and a fine-tuning might be needed to separate these scales.

Lagrangian is recovered by canonically normalizing the fields. For the case of a cubic interaction, the NDA coupling dictated by (3.4) is $\lambda \sim (\ell_5 \Lambda)^{1/2}$.

The gravitational cutoff Λ_{grav} is related to the AdS curvature k through (3.2) and (3.3). One may determine a similar relation between the matter cutoff Λ and k by considering the effective 4D interactions between specific KK modes. When expanding the 5D field in terms of canonically normalized 4D modes, $\Phi = kz \sum_n \tilde{f}_n(z) \phi_n(x)$, one finds that $f_n(1/\mu)$ is of order \sqrt{k} .³ Because KK modes are localized towards the IR brane, this implies that the order of magnitude of an effective 4D coupling between KK modes is obtained from the 5D coupling by multiplying by powers of \sqrt{k} and the warp factor $w = \mu/k$. For a given KK mode, the 4D NDA action is

$$S_{\text{KK}} = \frac{w^4 \Lambda^4}{\ell_4} \int d^4 x \hat{\mathcal{L}} \left[\hat{\phi}, \partial/(w\Lambda) \right] \quad (3.5)$$

following the same conventions of (3.4). Notice that the cutoff only appears through the warped down cutoff scale $w\Lambda = \tilde{\Lambda}$; we discuss this feature in Section 4.1.

Consider a general monomial interaction $\lambda_{5\text{D}} \Phi^n/n!$ in the 5D action with $n > 2$. 5D NDA, (3.4), reveals that the strong coupling coefficient is

$$\lambda_{5\text{D}} = \ell_5^{n/2-1} \Lambda^{5-3n/2}. \quad (3.6)$$

An interaction between n KK modes with $\mathcal{O}(1)$ dimensionless couplings is then

$$\lambda_{4\text{D}} \sim \ell_5^{n/2-1} \Lambda^{5-3n/2} k^{n/2-1} w^{4-n}. \quad (3.7)$$

On the other hand, the 4D NDA value for λ_4 is

$$\lambda_4 = \ell_4^{n/2-1} \Lambda^{4-n} w^{4-n}. \quad (3.8)$$

For the effective theory of KK modes to be valid, one must require the effective λ_4 in (3.7) to be smaller than or equal to its strong coupling estimate, (3.8). This implies

$$\Lambda > \frac{\ell_5}{\ell_4} k. \quad (3.9)$$

This universal relation arises because the \sqrt{k} and the loop factors have the same powers in the NDA estimates, which are in turn fixed by field counting. When (3.9) is not saturated, the effective 4D couplings of KK monomials are suppressed by powers of $(\ell_5 k/\ell_4 \Lambda)^{1/2}$ with respect to their strong coupling value. We now show that this systematic suppression factor is reminiscent of the large N suppression in the dual CFT.

3.3 AdS/CFT

One can understand the $(\ell_5 k/\ell_4 \Lambda)$ suppression in the framework of the AdS/CFT correspondence. To do so we need to consider the 5D theory using an appropriate variable, which is the value of the bulk field on the UV brane

$$\hat{\Phi}_0(x) \equiv \hat{\Phi}(X) \Big|_{\text{UV brane}}. \quad (3.10)$$

³ The KK mode normalization is $\int dz (kz)^{-1} \tilde{f}_n(z) \tilde{f}_m(z) = \delta_{mn}$. One has $\tilde{f}_m(z) = (kz)^{-1} f_m(z)$, where the f_m are introduced in Sec. 2.

Φ is the dimensionless bulk field in (3.4). The bulk field in the action is rewritten as $\hat{\Phi} = \hat{\Phi}_0 K$, where K is the classical field profile sourced by $\hat{\Phi}_0$. In terms of this holographic variable, the partition function takes the form $\int \mathcal{D}\hat{\Phi}_0 \exp\left(iS_5[\hat{\Phi}_0 K]\right)$, where S_5 is the 5D action for which the 5D NDA in (3.4) applies.

The leading term of the effective action in the semiclassical expansion is the classical holographic action

$$\Gamma_{\text{hol}} = \frac{\Lambda^5}{\ell_5} \int d^4x \mathcal{L}_{\text{hol}} \left[\hat{\Phi}_0, \partial/\Lambda \right] + \dots, \quad (3.11)$$

where the ellipses represent quantum terms that are irrelevant for our discussion. The Lagrangian \mathcal{L}_{hol} has dimension -1 . To recover a 4D NDA formulation as in (3.4), we need to introduce a dimensionless Lagrangian. From explicit calculation (see e.g. [4, 44]), the quadratic part of \mathcal{L}_{hol} , $\frac{1}{2}\hat{\Phi}_0 \Pi[\partial^2] \hat{\Phi}_0$, is proportional to the inverse of $\Delta_q(z_0, z_0)$ and contains an analytic part representing a 4D mode. Schematically, it is

$$\Pi[\partial^2] \sim -\frac{1}{k} \frac{\partial^2 + m_0^2}{\Lambda^2} + \dots \quad (3.12)$$

up to an $\mathcal{O}(1)$ coefficient. In the language of AdS/CFT, this is the kinetic term of the 4D source probing the CFT. The exact expression can be read directly from the propagator (4.9) and is not needed here.

We thus introduce the dimensionless Lagrangian $\frac{1}{k} \hat{\mathcal{L}}_{\text{hol}} = \mathcal{L}_{\text{hol}}$, such that the dimensionless source described in (3.12) is canonically normalized. The action now can be rearranged as

$$\Gamma_{\text{hol}} = \left(\frac{\ell_4 \Lambda}{\ell_5 k} \right) \frac{\Lambda^4}{\ell_4} \int d^4x \hat{\mathcal{L}}_{\text{hol}} \left[\hat{\Phi}_0, \partial/\Lambda \right] + \dots, \quad (3.13)$$

where we explicitly write the Λ^4/ℓ_4 factor appear in accordance with 4D NDA. The factor in parenthesis is the same suppression obtained in Section 3.2. From (3.11) it is clear that this factor systematically appears alongside \hbar in the semiclassical expansion of the holographic action.

We may now perform dimensional analysis on the canonically normalized holographic variable,

$$\Phi_0 = \left(\frac{\ell_4 \Lambda}{\ell_5 k} \right)^{1/2} \frac{\Lambda}{\ell_4^{1/2}} \hat{\Phi}_0. \quad (3.14)$$

Functional derivatives with respect to Φ_0 are suppressed as

$$\frac{\delta^n \Gamma_{\text{hol}}}{\delta \Phi_0(x_1) \delta \Phi_0(x_2) \dots} \propto \left(\frac{\ell_5 k}{\ell_4 \Lambda} \right)^{n/2-1} \quad (3.15)$$

at leading order. Hence by applying dimensional analysis at 5D and 4D levels in the holographic action, we have shown that a small parameter $(\sqrt{\ell_5 k}/\ell_4 \Lambda)$ systematically suppressing the interactions and controlled by the AdS curvature appears.

The AdS/CFT correspondence dictates that the above quantity reproduces the connected n -point functions of a conformal gauge theory with adjoint fields and large N . The main contribution to the correlator at large N is suppressed as [52, 53]

$$\langle \mathcal{OO} \dots \rangle_{\text{con}} \propto \frac{1}{N^{n-2}}. \quad (3.16)$$

with canonical normalization such that the two-point function does not scale with N . Comparing the AdS expression (3.15) and the CFT expression (3.16), we see that the suppression factor in AdS corresponds to the $1/N^2$ suppression of the CFT,

$$\frac{\ell_5 k}{\ell_4 \Lambda} \sim \frac{1}{N^2}. \quad (3.17)$$

We thus obtain a precise, field-theoretical version of the correspondence between the $1/N$ expansion in the CFT and the parameters of the AdS effective field theory. At fixed AdS curvature k , and i.e. fixed 't Hooft coupling, the $N \rightarrow \infty$ limit corresponds to the $\Lambda \rightarrow \infty$ limit. This sets all interactions to zero and therefore produces a free 5D theory. The relation (3.17), when put in the holographic action (3.13), gives $\Gamma_{\text{hol}} = N^2 \frac{\Lambda^4}{\ell_4} \int d^4 x \hat{\mathcal{L}}_{\text{hol}}$. The N^2 factor accompanying \hbar in this action is a hallmark feature of AdS/CFT [5].

3.4 Value of the Cubic Coupling

In this work we consider a scalar field, whose natural mass scale would be $\mathcal{O}(\Lambda)$, as reflected by NDA. While the NDA value of the cubic coupling is $\lambda \sim (\ell_5 \Lambda)^{1/2}$, for this manuscript we set it to a smaller value

$$\lambda \sim m_\Phi \frac{\ell_5^{1/2}}{\Lambda^{1/2}}. \quad (3.18)$$

This value is consistent with a bulk mass parametrically lower than Λ : the self-energy bubble diagram from λ gives a $\mathcal{O}(m_\Phi^2)$ contribution, in accordance with NDA. The λ coupling tends to zero in the free limit $\Lambda \rightarrow \infty$ (i.e. $N \rightarrow \infty$) as it should.

4 The Kaluza–Klein and Continuum Regimes of AdS

We study the behavior of the effective theory using the results of the free theory in Section 2 and the interaction strengths in Section 3. Quantum corrections from the bulk interactions ‘dress’ the bulk propagator and cause it to have qualitatively different behavior depending on the four-momentum, p . We show how these corrections separate the Kaluza–Klein and continuum regimes of a bulk scalar.

4.1 The Transition Scale

The homogeneity of AdS implies a homogenous 5D cutoff on proper distances smaller than $\Delta X \sim 1/\Lambda$. In the conformal coordinate system the cutoff is z -dependent with respect to the Minkowski distance $\sqrt{\eta_{\mu\nu} \Delta x^\mu \Delta x^\nu}$. This implies that while the 5D cutoff for an observer on the UV brane is Λ , the cutoff for an observer at position z in the bulk is warped down to $\Lambda/(kz)$.

One can see this from an EFT perspective: the effects of higher-dimensional operators in the action are enhanced by powers of z . For example, consider dressing the propagator with a higher derivative bilinear, $\Box(\partial_\mu \Phi)^2/\Lambda^2$ with an $\mathcal{O}(1)$ as dictated by NDA, see (3.4). This term dominates for

$$pz \gtrsim \Lambda/k. \quad (4.1)$$

For a fixed p , this implies that the EFT breaks down in the IR region of AdS, $z \gtrsim (\Lambda/k)/p$; see e.g. [30–32]. At values of z beyond this region, the cutoff is warped below the scale p . Propagation into this region of position-momentum space is forbidden in the EFT.

It follows that the theory also contains a scale

$$\tilde{\Lambda} = \Lambda \frac{\mu}{k}, \quad (4.2)$$

the warped down cutoff at the IR brane. At energies $p > \tilde{\Lambda}$, the correlation functions cannot know about the IR brane since it is in the region of position-momentum space hidden by the EFT validity condition (4.1). In short, for $p > \tilde{\Lambda}$ the IR brane is “outside of the EFT”.

This is a hint that the behavior of the theory undergoes a qualitative change at $\tilde{\Lambda}$. The IR brane imposes a boundary condition that leads to discrete KK modes. Thus for $p < \tilde{\Lambda}$, one can expect that the theory features KK modes. On the other hand, for $p > \tilde{\Lambda}$ the IR brane is outside the EFT, hence no KK modes should exist. Instead, an observer should see a continuum of states.

4.2 Dressed Propagator

The free propagator in (2.9) encodes narrow KK modes. It amounts to $\Lambda \rightarrow \infty$ or $N \rightarrow \infty$. The continuum behavior becomes apparent when one dresses the free propagator with quantum corrections.⁴ These quantum corrections resolve the poles in the free propagator with timelike momenta as they do in 4D Minkowski space. Including these effects corresponds to evaluating the leading $1/N^2$ effect on the propagator of the strongly coupled dual theory; in our case this is $1/N^2 \sim \lambda^2/k$.

We focus on bulk self-energy corrections from a cubic self-interaction. Brane-localized self-energies only modify the boundary conditions and are thus unimportant for our purposes. In contrast to the free propagator, the Green’s function equation for the dressed propagator satisfies

$$D_X \Delta(X, X') - \frac{1}{\sqrt{g}} \int dY \Pi(X, Y) \Delta(Y, X') = -\frac{i}{\sqrt{g}} \delta^{(5)}(X - X'), \quad (4.3)$$

where $i\Pi(X, Y)$ are 1PI insertions that dress the propagator. In our case, the leading $i\Pi$ insertion is induced by the scalar bubble induced by the $\lambda\Phi^3$ interaction. We are interested only in the imaginary part of the self-energy, which is finite.

A calculation of $i\Pi(X, Y)$ is performed analytically in [32] with self-consistent approximations in the limit of strong coupling and moderate bulk masses $\alpha = \mathcal{O}(1)$. One of the tricks for the analytical estimate is to expand the non-local self-energy as a series of local insertions, which amounts to a ∂_z expansion. Using this method, we estimate of the contribution from the $|p| > 1/z_<$ regime. The imaginary part of the 1-loop bubble induces a shift of p ,

$$\Delta_p^{\text{dressed}}(z, z') \sim \Delta_{p(1-ic)}^{\text{free}}(z, z') \quad c \sim a \frac{\lambda^2}{\ell_5 k}, \quad (4.4)$$

where c is loop-induced and estimated to have $a \sim \mathcal{O}(1/10)$ with a large uncertainty. Using the NDA value of λ in (3.18) and taking $m_\Phi = \mathcal{O}(k)$, one finds $c \sim ak/\Lambda \sim a/(\pi N^2)$. The $|p| > 1/z_<$

⁴ The exact calculation of diagrams in AdS has recently been an intense topic of research, see e.g. [54–59] for loop-level diagrams and [60–63] for developments in position-momentum space. Throughout this paper we instead use approximate propagators.

regime provides a larger contribution to a than the result previously presented from the $|p| < 1/z_>$ regime [32]. This extends the validity of our calculations to weaker coupling, hence allowing large N . A self-consistent numerical solution to the integro-differential equation of motion, (4.3), may be required to obtain the general dressed propagator. We leave this for future work.

4.3 The Two Regimes

The self-energy dressing of the propagator presents distinct Kaluza–Klein and continuum regimes. The poles of the free propagator are set by zeros of its denominator. For momenta much larger than the IR brane scale, $p \gg \mu$, the asymptotic form of the Bessel functions lead to a propagator that is approximately proportional to

$$\Delta_p(z, z') \propto \frac{1}{\sin\left(\frac{p}{\mu} - \frac{\pi}{4}(1 + 2\alpha)\right)}. \quad (4.5)$$

The effect of the dressing, (4.4), softens the poles and causes them to merge at a scale

$$p \sim \frac{\mu}{c} \sim \frac{\tilde{\Lambda}}{a}. \quad (4.6)$$

Above this scale the propagator describes a continuum rather than distinct Kaluza–Klein modes. Thus we observe that the dressing of the propagator reaffirms the existence of distinct KK and continuum regimes separated by a transition scale controlled by $\tilde{\Lambda} = (\mu/\Lambda)k$. Let us comment further on both sides of the transition.

4.4 Kaluza–Klein Regime: $p < \tilde{\Lambda}$

For momenta less than the transition scale $\tilde{\Lambda}$, a probe is sensitive to the physics of the IR brane. The IR brane provides a boundary condition for the bulk equation of motion and hence imposes a discrete spectrum of KK modes. These modes may be narrow. However, as the KK mass approaches the transition scale, the KK modes must merge to form a continuum. To see this, one may use the full form of the dressed KK propagator from (4.3). This propagator may be written

$$\Delta_q(z, z') = i \mathbf{f}(z) \cdot [\mathbf{D}^{-1} + i \text{Im } \mathbf{\Pi}]^{-1} \cdot \mathbf{f}(z') \quad (4.7)$$

where

$$i \mathbf{\Pi} \equiv \int du \int dv i \Pi(u, v) \mathbf{f}(u) \otimes \mathbf{f}(v). \quad (4.8)$$

The imaginary part of $\mathbf{\Pi}$ gives rise to a “width matrix” for the KK resonances. Critically, $\text{Im } \mathbf{\Pi}$ is *not* diagonal: the KK modes mix due to this non-diagonal, imaginary contribution to the mass matrix. The KK modes may merge into a continuum either because they become broad, or because of the mixing induced by $\text{Im } \mathbf{\Pi}$. This property of the AdS propagator is suggestive of how heavy glueballs in the strongly-coupled dual tend to merge near the $\tilde{\Lambda}$ cutoff.

4.5 Continuum Regime: $p > \tilde{\Lambda}$

When p is above the transition scale, $\tilde{\Lambda}$, the oscillating pieces of the propagator are smoothed. Within this regime, the endpoints of the propagator define additional scales for which the propagator realizes different behavior.

Continuum regime, low momentum. In the continuum regime with low momentum, $|p| > \tilde{\Lambda}$ and $|p| < z_{>}^{-1}$, the propagator is

$$\Delta_p(z, z') \approx \Delta_{\text{UV}} + \Delta_{\text{heavy}} + \Delta_{\text{light}} , \quad (4.9)$$

where the pieces are

$$\Delta_{\text{UV}} = i \frac{(b_{\text{UV}} + 2\alpha)(kz)^{2-\alpha}(kz')^{2-\alpha}}{\alpha(p^2/(\alpha-1)k + 2b_{\text{UV}}k)} \quad \Delta_{\text{heavy}} = -i \frac{(kz)^2(kz')^2}{2\alpha k} \left(\frac{z_{<}}{z_{>}} \right)^{2\alpha} \quad (4.10)$$

$$\Delta_{\text{light}} = -i \frac{\Gamma(-\alpha)(kz)^2(kz')^2}{\Gamma(\alpha+1)2b_{\text{UV}}^2 k} g(z_{<})g(z_{>}) \left(\frac{-p^2}{4k^2} \right)^\alpha \quad g(z) = \frac{b_{\text{UV}} + 2\alpha}{(zk)^\alpha} - b_{\text{UV}}(zk)^\alpha . \quad (4.11)$$

Notice that the dependence on the μ parameter has dropped this expression. This is a manifestation of the propagator's agnosticism of the IR brane in this regime. Conversely, this implies that when varying p from UV scales to IR scales, the IR brane is effectively emergent when p drops below $\tilde{\Lambda}$.

The content of each term in (4.9) is also instructive. The first term, Δ_{UV} represents a 4D mode localized near the UV brane.⁵ This 4D mode is assumed to be very heavy, $b_{\text{UV}} = \mathcal{O}(1)$, such that it does not play a role in the processes of in this manuscript. The second term, Δ_{heavy} is analytic and encodes the collective effect of heavy KK modes. The third term, Δ_{light} is nonanalytic and encodes the collective effect of light modes.

Continuum regime, high momentum. In the continuum regime with high momentum, $|p| > \tilde{\Lambda}$ and $z_{>}^{-1} < |p| < z_{<}^{-1}$, the numerator of the propagator oscillates:

$$\Delta_p(z, z') \propto \frac{\cos(p\mu^{-1} - pz_{>})}{\cos(p\mu^{-1} + \varphi_-)} \times \begin{cases} 1 & \text{for } z_{>}^{-1} < p < z_{<}^{-1} \\ \cos(pz_{<} - \varphi_+) & \text{for } p > z_{<}^{-1} , \end{cases} \quad (4.12)$$

where we have written phase shifts as $\varphi_{\pm} = \pi(1 \pm 2\alpha)/4$. Upon dressing, the non-oscillatory part of the propagator in this region scales as

$$\Delta_p(z, z') \sim \begin{cases} e^{-|p|z_{>}} & \text{for } p_{\mu} \text{ spacelike} \\ e^{-cpz_{>}} & \text{for } p_{\mu} \text{ timelike} \end{cases} . \quad (4.13)$$

This is an important feature: the IR region of AdS is opaque to propagation for both spacelike and timelike momenta. The regions of opacity are somewhat different—the suppression for spacelike momentum occurs at $z \sim 1/|p|$, while the suppression for timelike momentum occurs at $z \sim 1/cp$. Substituting in c , we see that the suppression in the timelike regime occurs for

$$pz_{>} \gtrsim \frac{\Lambda}{ak} . \quad (4.14)$$

This behavior is similar to the region of EFT breaking in (4.1). Therefore the opacity of the space effectively censors the region where the EFT breaks down. This behavior is qualitatively predicted in [30]. For the specific case with an endpoint on the IR brane, $z_{>} = 1/\mu$, the opacity threshold (4.14) is the same as the scale at which the Kaluza–Klein poles disappear, (4.6). The two effects are, of course, closely related: the poles vanish precisely when the IR brane becomes opaque to the propagator.

⁵In our convention, the 4D mode squared mass is positive for negative b_{UV} .

In the continuum regime, KK modes are not appropriate variables to describe the theory because the f_n profiles fall into a spacetime region where the EFT breaks down, (4.1). Instead, the meaningful variables are those localized on the UV brane. These remain in the theory up to the ultimate cutoff $p \sim \Lambda$. This was already observed in [31] from EFT considerations, and is completely consistent with the holographic formalism needed for AdS/CFT.

4.6 CFT Interpretation

It is interesting to consider the CFT interpretation of the transition scale using $\Lambda/k \sim N^2 \ell_5/\ell_4$ as established in (3.17). In this discussion we will often use $\ell_5/\ell_4 \sim \pi$. Interactions vanish in the $\Lambda \rightarrow \infty$ limit. AdS theory in that limit thus contains an infinite tower of free, stable KK modes. This is the AdS manifestation of the infinite tower of stable glueballs when $N \rightarrow \infty$.

For finite N , the transition scale is

$$\tilde{\Lambda} \sim N^2 \pi \mu. \quad (4.15)$$

The scale controlling the mass of the KK modes, $\pi \mu$ appears. The KK masses grow linearly, hence the transition is reached around the N^2 -th KK masses. In the dual CFT theory this corresponds to the N^2 -th glueball.

The $\tilde{\Lambda}$ scale would be the cutoff of the glueball EFT. Does the value (4.15) make sense from the gauge theory side? Recall that the large- N theory contains, in principle, many glueballs at low energy. It is thus described by an EFT containing many species. The interactions between glueballs are set by the $\tilde{\Lambda}$ scale and suppressed by powers of $1/N$. In the loop diagrams, such suppression is compensated by the multiplicity of glueballs. Once N^2 glueballs are introduced, the cutoff of the EFT becomes $\tilde{\Lambda}$. This feature can be seen by using 4D NDA applied to the glueball theory with arbitrary number of species N_s and $D = 4$. The prefactor of (3.4) is

$$\frac{N_s \tilde{\Lambda}^4}{N^2 \ell_4}, \quad (4.16)$$

which indicates strong coupling when the number of glueballs N_s is of order N^2 . This paints a consistent picture: the N^2 modes of the KK regime in AdS match the N^2 glueballs of the gauge theory.

These considerations are only about the number of species and do not tell us about glueball masses. However we also know that an infinite tower of glueballs is needed to reproduce the logarithmic momentum dependence of the correlator between gauge currents [52]. At finite N , the width over mass ratio of the n^{th} glueball is expected to grow as $\Gamma_n/m_n \sim n/N^2$ because it can decay into lighter glueballs via $1/N$ suppressed cubic vertices. Hence the glueballs tend to become broad at $n \sim N^2$, which signals the transition to a continuum. Since there is a tower of glueballs, the cutoff of the glueball EFT has to be around $\tilde{\Lambda} \sim m_{N^2}$. This matches the picture obtained on the the AdS side in (4.15), where the N^2 -th KK mode is indeed of order of $\tilde{\Lambda}$.

5 Cascade Decays in the Continuum Regime

Section 4 presents a global picture of timelike propagation in a slice of AdS and the emergence of the IR brane. However, an important aspect remains to be studied. Bulk interactions are tied to the emergence of the IR brane via opacity and the EFT cutoff. These interactions induce opacity

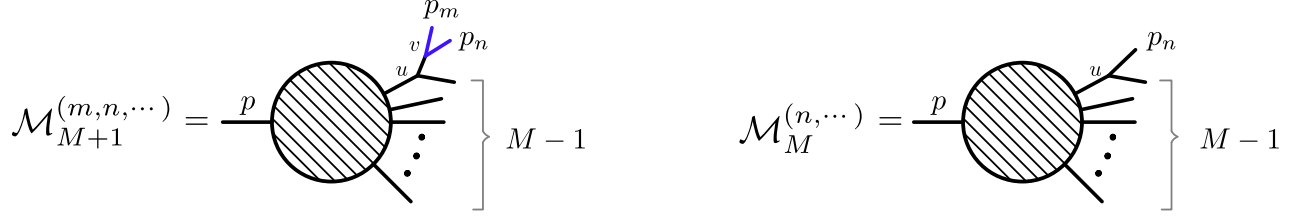


Figure 1: The cascade decay amplitudes. u and v are coordinates in the z direction. In our recursive approach, we relate the integrated square amplitude of the left diagram to that of the right diagram.

through the self-energy, which is tied to the decays of bulk excitations by unitarity cuts. This may appear as a loophole to emergence; an observer in the UV may transmit high-energy information to the IR if the field excitation undergoes a cascade decay into a multitude of lower-energy particles that each evade the opacity suppression. We diagnose this process in this section and discuss implications in Section 6.

The properties of cascade decays initiated in the KK regime are fairly well-understood and are summarized in Section 6.1. We instead focus on the cascade decays starting in the continuum regime. This regime is always present unless interactions are removed ($\Lambda \rightarrow \infty$). Furthermore, in the strong coupling limit $\Lambda \sim k$, there is essentially no KK regime and all propagation is in the continuum regime. We seek to determine the overall shape and the total probability for a cascade decay event to occur in the continuum regime.

The bulk of AdS does not permit asymptotic states or a conventional S -matrix (see e.g. [64,65]). However the 4D modes localized on the branes, which have a 4D Minkowski metric, can provide usual asymptotic states. We thus consider decays that are initiated on the UV brane. The decay may end back on the UV brane or reach asymptotic states on the IR brane. It can also end in narrow KK modes which are effectively asymptotic states in the limit of the 4D narrow width approximation.

5.1 The Decay Process

The explicit evaluation of a generic decay diagrams with an arbitrary number of legs is, in principle, challenging because there are many phase space and position integrals to perform over a non-trivial integrand. However, it turns out that a recursive approach can be adopted based on simplifying approximations. We build on this approach to estimate the total rate for a generic decay.

For intermediate steps in this calculation, it is convenient to formally write the final states as KK modes, even if the corresponding momenta are in the continuum regime. Sums over KK modes may then be re-expressed in terms of the closed form propagator at the end of the calculation.

Measurable event rates such, as cross sections and decay widths, depend on the integral of the squared amplitude over phase space. To emphasize that our approach does not depend on how the continuum is created, we work at the level of this integrated square amplitude, denoted as P_M . For the diagram in Fig. 1 with $M + 1$ final states,

$$P_{M+1} \equiv \int \sum_{\text{FS}(M+1)} |\mathcal{M}_{M+1}|^2 (2\pi)^4 d\Phi_{M+1}. \quad (5.1)$$

The sum over $\text{FS}(M + 1)$ is shorthand for a sum over all possible combinations of $(M+1)$ KK modes that are kinematically allowed final states. $d\Phi_{M+1}$ is the volume element of the $(M + 1)$ -body

Lorentz-invariant phase space [66]. We label specific final state KK numbers and four-momenta as m , p_m and n , p_n . The amplitude for a given set of final state KK modes is expressed as

$$\mathcal{M}_{M+1}^{(m,n,\dots)} = \int du \mathcal{I}_M^{(\dots)}(u) \int_{1/k}^{1/\mu} dv \frac{\lambda \Delta_q(u, v)}{(kv)^5} f_m(v) f_n(v) . \quad (5.2)$$

$\mathcal{I}_M^{(\dots)}(u)$ is the amplitude that has been amputated just before the propagator that produces the m and n modes, see Fig. 1.

The \mathcal{M}_M amplitude, shown on the right-hand side of Fig. 1, is

$$\mathcal{M}_M^{(n,\dots)} = \int du \mathcal{I}_M^{(\dots)}(u) f_n(u). \quad (5.3)$$

The corresponding integrated square amplitude is

$$P_M \equiv \int \sum_{FS(M)} |\mathcal{M}_M|^2 (2\pi)^4 d\Phi_M . \quad (5.4)$$

We now relate P_{M+1} to P_M .

5.2 Recursion Relation

Propagators with timelike momentum are suppressed beyond $z_{>} \sim 1/(cp)$, as seen in (4.13). We assume for simplicity that $c \sim 1$. This implies that our evaluation assumes nearly strong coupling, i.e. Λ is not far from k . Following this, the position integrals effectively have no support beyond $z \sim 1/p$. Note that this is equivalent to only considering contributions from the $\mu < |p| < z_{>}^{-1}$ region of position-momentum space, see (4.9).

We have numerically evaluated contributions from the $|p| > z_{>}^{-1}$ regions and found that they tend to be smaller or of the same order as the results from this section for c near unity. These contributions can be somewhat larger for smaller c , though a detailed analysis is beyond the scope of this manuscript.

We square the amplitude and write sums on KK modes as integrals over the propagator using (2.13). In the continuum regime, only the third term of the continuum propagator in (4.9) contributes to the contour integral because it carries a branch cut. By deforming the contour to fit snugly around the branch cut, we determine that

$$\sum_{n=0}^{\tilde{n}} U(m_n) f_n(z) f_n(z') = -\frac{1}{2\pi} \oint_{\mathcal{C}[\tilde{n}]} dq^2 U(q^2) \Delta_q(z, z') = \frac{-1}{2\pi} \int_0^{m_{\tilde{n}}^2} dq^2 U(q^2) \text{Disc}[\Delta_q(z, z')] . \quad (5.5)$$

In terms of the propagator, P_{M+1} then reads

$$P_{M+1} = 4\pi^2 \sum_{FS(M-1)} \int d\Phi_{M+1} \int du \int du' \mathcal{I}_M(u) \mathcal{I}_M^*(u') \int_{1/k}^{1/q} dv \int_{1/k}^{1/q} dv' \frac{\lambda^2 \Delta_q(u, v)}{(kv)^5 (kv')^5} \times \quad (5.6)$$

$$\int dp_1^2 \text{Disc}[\Delta_{p_1}(v, v')] \int dp_2^2 \text{Disc}[\Delta_{p_2}(v, v')] .$$

The integrals over the p_1^2, p_2^2 variables implement the sum over KK modes in (5.5). We break up the phase space using the standard recursion relation, see e.g. [66],

$$d\Phi_{M+1} = d\Phi_2(q; p_1, p_2) d\Phi_M (2\pi)^3 dq^2. \quad (5.7)$$

The integrands (5.6) carry positive powers of v and v' so that the $dv dv'$ integrand is largest the upper limit, $v, v' \sim 1/q$. Because q is the momentum flowing through the parent this implies that the cascade decay progresses slowly towards the IR region.

$$P_{M+1} = (2\pi)^4 \sum_{\text{FS}(M-1)} \int d\Phi_M \int du \int du' \mathcal{I}_M(u) \mathcal{I}_M^*(u') \int_{1/k}^{1/q} dv \int_{1/k}^{1/q} dv' \frac{\lambda^2 \Delta_q(u, v)}{(kv)^5 (kv')^5} \times \quad (5.8)$$

$$\int dp_1^2 \text{Disc}[\Delta_{p_1}(v, v')] \int dp_2^2 \text{Disc}[\Delta_{p_2}(v, v')] \int \frac{dq^2}{64\pi^4 q^2} K(q, p_1, p_2).$$

Here $K(q, p_1, p_2)$ is the 2-body kinematic factor,

$$K(q, p_1, p_2)^2 \equiv [q^2 - (p_1 + p_2)^2] [q^2 - (p_1 - p_2)^2]. \quad (5.9)$$

We approximate the integrals over p_1^2 and p_2^2 as

$$\int_0^q dp_1 \int_0^{q-p_1} dp_2 p_1^{2\alpha+1} p_2^{2\alpha+1} K(q, p_1, p_2) \approx \int_0^{q/2} dp_1 \int_0^{q/2} dp_2 p_1^{2\alpha+1} p_2^{2\alpha+1} q^2. \quad (5.10)$$

This approximation introduces a small amount that depends on α .⁶ Note that the dominant contribution to the integral in (5.10) comes from the region near the upper limit. This indicates that the continua tend to decay near kinematic threshold. The cascades gives rise to soft spherical final states, in accordance with former results from both gravity and CFT sides.

Integrating over p_1^2, p_2^2, v , and v' , we have

$$P_{M+1} = C_\alpha \sum_{\text{FS}(M-1)} (2\pi)^4 \int d\Phi_M \int \frac{dq^2}{k} \left(\frac{q}{k}\right)^{2\alpha} \int du \int du' \mathcal{I}_M(u) \mathcal{I}_M^*(u') (ku)^{2+\alpha} (ku')^{2+\alpha}, \quad (5.11)$$

where the constant prefactor is

$$C_\alpha = \frac{8^{4(1-\alpha)} \lambda^2}{\alpha^4 \pi^4 k} \left(\frac{\Gamma(1-\alpha) \sin(\pi\alpha)}{\Gamma(1+\alpha)} \right)^2 \frac{|(2+3\alpha)4^\alpha - (\alpha+2) \frac{\Gamma(1-\alpha)}{\Gamma(1+\alpha)} e^{i\alpha\pi}|^2}{(2+3\alpha)^2 (2+\alpha)^2 (1+\alpha)^2}. \quad (5.12)$$

One may replace the dq^2 in favor of a sum over the continuum of KK final states by applying (5.5). This yields a recursion relation

$$P_{M+1} = r \int \sum_{\text{FS}(M)} \left| \int du \mathcal{I}_M(u) f_n(u) \right|^2 (2\pi)^4 d\Phi_n = r P_M. \quad (5.13)$$

The fact that one obtains a simple relation is a consequence of the integrand having a specific momentum dependence and is nontrivial. This relation is clearly useful since it can be used to give an estimate of a total rate with arbitrary number of legs.

The recursion coefficient r is given by

$$r \equiv \frac{\lambda^2}{k} \frac{1}{1024^{1+\alpha}} \frac{1}{2\pi^3 \alpha^3} \left(\frac{|(2+3\alpha)4^\alpha - (2+\alpha) \frac{\Gamma(1-\alpha)}{\Gamma(1+\alpha)} e^{i\alpha\pi}|^2}{(2+3\alpha)^2 (2+\alpha)^2 (1+\alpha)^2} \right) \frac{\Gamma(1-\alpha) \sin(\pi\alpha)}{\Gamma(1+\alpha)}. \quad (5.14)$$

Even for the strongly coupled case, $\lambda^2 \sim \ell_5 k$, this coefficient is much smaller than one.

⁶The error monotonically increases from $\sim 25\%$ for α near 0 to $\sim 30\%$ for α near 1.

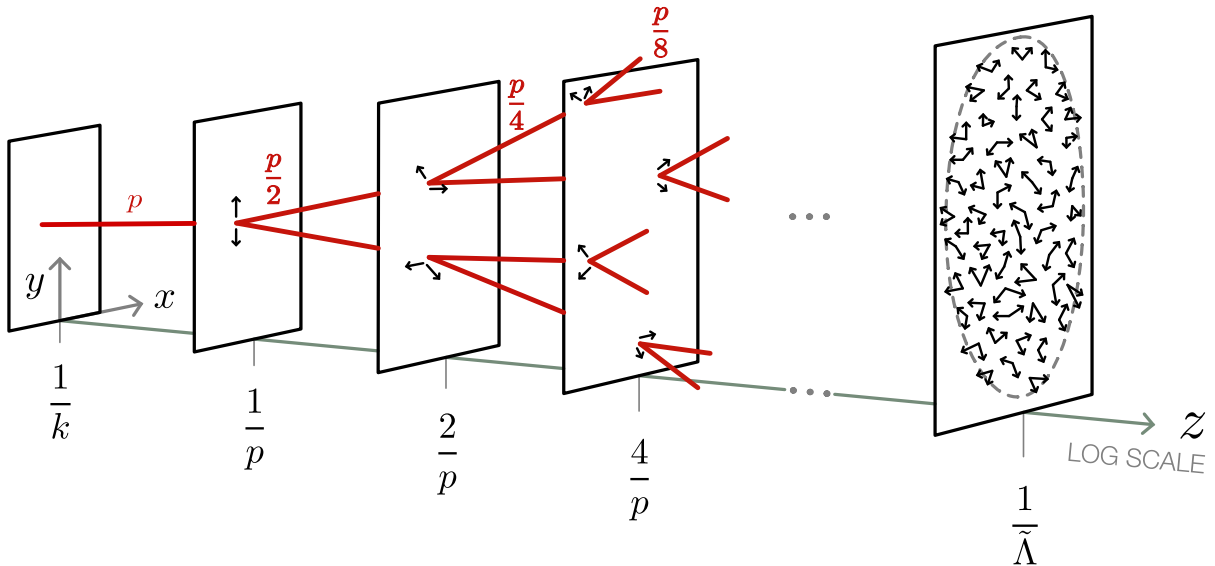


Figure 2: A typical field-theoretical soft bomb event in AdS_5 in the continuum regime $p > \tilde{\Lambda}$. The rate for such an event to occur is exponentially suppressed.

6 Soft Bombs and the Emergence of the IR Brane

The recursion relation (5.14) allows us to study the qualitative features of a complete cascade decay event. An event initiated on the UV brane with timelike momentum $P > \tilde{\Lambda}$ starts in the continuum regime and decays as a cascade of continua. This decay eventually reaches the KK regime.

6.1 Shape

The differential event rate—the integrand in the expression for P_M —determines the most likely configurations in phase space. The phase space approximation (5.10) shows that decays tend to occur near threshold with final momenta evenly split between the offspring. The event thus tends to be soft and spherical. This confirms the *soft bomb* picture obtained in the KK regime [29], in string calculations (see e.g. [23]) and in the gauge theory dual [25, 28].

The integrand in (5.8) shows that vertices tend to occur at $z \sim 1/p$ where p is the momentum of the parent continuum. There is a sense of progression in the fifth dimension: the cascade decay proceeds from the UV to the IR with each offspring moving further into the IR than its parent.

Let p_f be the average momentum of states after some number of branchings. The soft bomb then leaves the continuum regime and enters the KK regime at $p_f \sim \tilde{\Lambda}$. This is roughly the scale at which the KK modes become narrow. These features are summarized in Fig. 2.

6.2 Total Rate

The soft bomb enters the regime of narrow KK modes when the constituents have average momenta $p_f \sim \tilde{\Lambda}$. At this scale, the narrow width approximation is valid and the recursion (5.13) halts because subsequent decays factorize. This highlights a key feature of the continuum regime in contrast to the KK regime: the phase space suppression in cascade decays is not compensated by

narrow poles due to the breakdown of the narrow width approximation. Thus the rate of long cascade decays are suppressed by powers of the recursion coefficient r in (5.14).

One may estimate the total rate of cascade decays using the recursion approximation (5.13). A continuum cascade initiated with momentum P stops at momentum $p_f \sim \tilde{\Lambda}$. Assuming an equal split of momenta among a total of M offspring gives

$$M \sim P/\tilde{\Lambda} . \quad (6.1)$$

The recursion relation (5.13) shows that the rate is suppressed by r^{M-1} .

$$P_M \sim r^{P/\tilde{\Lambda}} . \quad (6.2)$$

Since $r \ll 1$, the soft bomb is exponentially suppressed as a function of initial timelike momenta in the continuum regime $P > \tilde{\Lambda}$.

6.3 Discussion

The suppression of the soft bomb rate in the continuum regime completes our picture of quantum field theory in AdS for timelike momenta. Notice in the KK regime there is no such suppression of the soft bomb rate.

Consider, for example, a UV-localized field φ that couples to the bulk scalar, Φ . The collision of two φ states can induce a cascade decay $\varphi\varphi \rightarrow \Phi \rightarrow \Phi\Phi \rightarrow \dots$. When the center-of-mass four-momentum is in the KK regime, $P < \tilde{\Lambda}$, the event rate is determined by the $\varphi\varphi \rightarrow \Phi^{(n)}$ amplitude to create an on-shell KK mode $\Phi^{(n)}$ with mass $m_n \sim P$. In contrast, in the continuum regime, $P > \tilde{\Lambda}$, the cascade is initiated with 5D continua that have no poles and thus no notion of being on-shell. Narrow KK modes only appear after the cascade has produced enough offspring for the typical momentum to drop below $\tilde{\Lambda}$. The amplitude required to calculate includes the entire cascade up to, and including, the first narrow KK modes. The rate for a cascade in the continuum regime is suppressed with respect to that in the KK regime by the tiny factor $r^{P/\tilde{\Lambda}}$ in (6.2).

The suppression of the soft bomb rate in the continuum regime implies that the theory in this regime truly does not know about the IR brane. Cascade decays are rather forced to end back into the UV-brane localized states. The observables—including cascade decays—in this regime of the theory can be equivalently obtained in AdS background with no IR brane. Notice such regime can be exactly described by an appropriate CFT model as dictated by the AdS/CFT correspondence. Conversely, the IR brane—and the fields and operators localized on it—appear only in the IR at $p \sim \tilde{\Lambda}$ and are thus effectively emergent.

6.4 Asymptotically AdS backgrounds

Our study focuses on a slice of pure AdS with no departure from AdS in the IR region. The qualitative features of our results can apply to models whose backgrounds are deformed in the IR. One kind of model is the slice of AdS stabilized by the Goldberger–Wise mechanism. This produces a non-negligible backreaction of the metric near the IR brane. Another class of model are those where the metric develops a naked curvature singularity in the IR—the soft-wall models, see *e.g.* [12–14, 17, 19, 20] for some points of entry in the literature. Such models are typically asymptotically AdS towards the UV brane, with the IR deformation becoming relevant near the IR brane/singularity.

One can apply the reasoning of Section 4 to these models. By dimensional analysis, there is some typical scale $\hat{\mu}$ associated with the IR region. A transition scale $\Lambda\hat{\mu}/k$ thus also exists, above which the IR region should drop from the correlation functions if the EFT is to remain under control.

More quantitatively, one can integrate out the IR region and encapsulate it into an effective IR brane with non-trivial form factors localized on it [20]. This holographic projection of the IR region demonstrates that the two regimes can indeed be meaningfully separated. The effective IR brane contains the details of the model-dependent KK regime. Since the bulk is pure AdS, our results from Sections 2-6 apply. This immediately shows that at high enough p , the effective IR brane leaves the theory, leaving thus a (quasi-)AdS continuum regime like the one described in this paper. Conversely, when decreasing p , the deviation from AdS gradually emerges from the viewpoint of a UV-brane observer.

6.5 Holographic Dark Sector

The soft bomb suppression rate has phenomenological implications for theories where a dark (or hidden) sector is confined to the IR brane and the Standard Model is confined to the UV brane, as recently proposed in [33]. Suppose, for concreteness, that the decay chain ends in stable IR brane particles that could naturally be identified with dark matter.

A standard way to search for dark matter at colliders is to look for missing energy signatures. In our holographic dark sector scenario, the suppression of the cascade decay rate in the continuum regime implies that the missing energy spectrum should vanish around the $\tilde{\Lambda}$ scale. This characteristic of the holographic dark sector framework is completely distinct from standard 4D dark sectors.

Another standard constraint on dark sectors with light states that couple to the Standard Model is stellar cooling from the emission of dark states. In the holographic dark sector scenario, stars emit KK modes with narrow widths when the temperature of the star is roughly between μ and $\tilde{\Lambda}$. In contrast, if the star is hotter than $\tilde{\Lambda}$, the center-of-mass energy for dark state production is typically in the continuum regime. The emission rates then become exponentially suppressed and there is effectively no anomalous star cooling. Compared to standard 4D dark sector models, our scenario introduces a momentum dependence that can evade stellar cooling bounds. This mechanism may be exploited to create models with a light mediator that couple to the Standard Model but completely evade stellar cooling bounds due to the momentum-dependence of the suppression to the soft bomb rate.

These effects are very interesting from a phenomenological viewpoint: they may alleviate experimental constraints and change the experimental complementarity of dark matter searches. The direct detection signatures of this type of framework are studied in [39], which also discusses some qualitative differences of timelike correlation functions in models of near-conformal sectors. We explore these effects in upcoming phenomenological studies.

7 Conclusion

We revisit the behaviour of an effective theory of interacting matter fields in a slice of AdS_5 . We study new features induced by bulk interactions for timelike four-momenta. These correspond to including the leading $1/N^2$ effects in the strongly coupled dual theory. We show using dimensional analysis that there is a transition scale, $\tilde{\Lambda}$, above which bulk propagators lose contact with the

IR brane because the latter falls beyond the domain of validity of the effective theory. The scale separates the Kaluza–Klein and continuum regimes of the bulk propagator. The continuum regime would be absent if interactions were not taken into account. Conversely the continuum regime is the only one present in the limit of strong interactions.

For timelike momenta the transition between the KK and continuum regimes occurs because the propagator is dressed by bulk interactions, a leading $1/N^2$ effect. This induces an exponential suppression of the propagator in the region where the EFT would become invalid. This censorship property was qualitatively predicted in [30]. Our treatment invokes approximations to loop integrals; more details of opacity in AdS may be better elucidated with future calculational developments.

In the CFT dual, the existence of the transition scale corresponds to the fact that the effective theory of glueballs cannot contain infinitely many species. It becomes strongly coupled if more than approximately N^2 glueballs are included in the spectrum. Beyond the transition scale, a gauge theory with no mass gap should appear. This is indeed what we demonstrate in the AdS theory.

For timelike bulk propagators, the IR brane is effectively absent when $p > \tilde{\Lambda}$. However, cascade decays could allow correlators with energy beyond $\tilde{\Lambda}$ to be sensitive to the IR brane because the momentum is split between many offspring states. We therefore study cascade decays to better understand the notion of IR brane emergence. We focus on a scalar with a bulk cubic interaction and investigate the squared matrix element integrated over final states that are the main ingredients of observable event rates. A recursion relation between cascades of different branching depth is valid for a range of momenta. In this regime, we estimate of the rate for arbitrarily deep cascade.

We have checked that contributions from other effects are subleading. These include direct decays into an IR brane localized state or into light KK modes via a tower of off-shell KK modes. We found that the contribution from the region in which the propagator is exponentially suppressed may be of the same order, but that this does not change our conclusions.

The cascade decay calculation provides a picture of soft bombs in the continuum regime of AdS. We find that the shape of the cascade events tend to be soft and spherical in the 4D Minkowski slices. This is because the momentum tends to be split evenly between states near-threshold, which matches previous results for the CFT dual. Along the fifth dimension, the decays tend to occur near the region $z \sim 1/p$ where p is the parent four-momentum. Therefore the soft bomb diagram grows in the Minkowski direction and slowly progresses towards the IR. Once the typical momentum of the offspring reaches $\tilde{\Lambda}$, the soft bomb enters the KK regime.

While there is no diagrammatic change between the KK and continuum regimes, the crucial change occurs in the behaviour of the propagators. In the KK regime, the narrow width approximation applies, such that amplitudes giving the soft bomb rate can effectively be cut. In the continuum regime the propagators do not have poles and the event cannot be cut before reaching the KK regime. The phase space factor associated with each of the final states accumulate and the soft bomb rate in the continuum regime acquires an exponential suppression. It follows that the continuum regime can be described by a high-energy effective theory with no IR brane. In other words, the IR brane and its operators effectively emerge in the IR at the energy scale $E \sim \tilde{\Lambda}$. We expect that the same conclusions qualitatively apply to asymptotically AdS backgrounds with a metric deformation in the IR region, such as soft-wall models.

These features can lead to new possibilities for physics beyond the Standard Model, as already pointed out in [33]. In particular holographic dark sector scenarios may have bulk fields that mediate interactions between a UV-brane localized Standard Model and IR brane dark states that are emergent. This implies that a light dark particle can be invisible at high energy experiments.

For instance, bounds from stellar cooling or missing energy searches may be alleviated if the dark particles are light enough. The many phenomenological consequences of an emergent dark sector require further studies.

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